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Common final exam for Math 118, December 15, 2021.

YOUR NAME: ____________________ SECTION: ____________________

INSTRUCTOR: ____________________

DID YOU HAVE ANOTHER EXAM 5:30-7:30 TODAY? ____________________

Directions:

- Print your name, section number and your instructor's name on this page in the space provided.
- This exam has 12 questions. Please check that your exam is complete.
- You have two hours to complete this exam. It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator and the list of equations provided by the Department.
- When using decimals round your answers till three decimal places.
- Use of notes, books, any internet resources and electronic devices is NOT allowed.
- You may not communicate with anyone besides the instructor during this exam.

Problem	Score
1	/12
2	/9
3	/8
4	/6
5	/6
6	/12
7	/8
8	/6
9	/8
10	/8
11	/5
12	/12

Good luck!

1. (Points: 12) The number of asthma sufferers in the world was about 84 million in 1990 and 334 million in 2012. Let N represent the number of asthma sufferers (in millions) worldwide t years after 1990.

   (a) Model N as a linear function of year t after 1990.

   (b) Model N as an exponential function of year t after 1990.

   (c) How many asthma sufferers are predicted worldwide in 2020 with the linear model?

   (d) How many asthma sufferers are predicted worldwide in 2020 with the exponential model?

2. (Points: 9) Rank the following three bank-deposit options from best to worst.

   (a) Bank A: nominal rate 2% compounded daily

   (b) Bank B: nominal rate 2.1% compounded monthly

   (c) Bank C: nominal rate 2.05% compounded continuously

3. (Points: 8) Technetium-99m is a radioactive substance used to diagnose brain diseases. Its half-life is approximately 6 hours. Initially you have 200 mg of technetium-99m.

   (a) Write an equation that gives the amount of the substance remaining after t hours.

   (b) Determine the number of hours needed for your sample to decay to 120 mg.

4. (Points: 6) What is the long-run behavior of the function given below?

   (a) $x \to \infty, \quad y = \dfrac{x(x+6)(x-9)}{4+x^2} \longrightarrow$

   (b) $x \to -\infty, \quad y = \dfrac{x(x+6)(x-9)}{4+x^2} \longrightarrow$

5. (Points: 6)

(a) Find the angle between $0°$ and $360°$ (but not $240°$) that has the same cosine as $240°$.

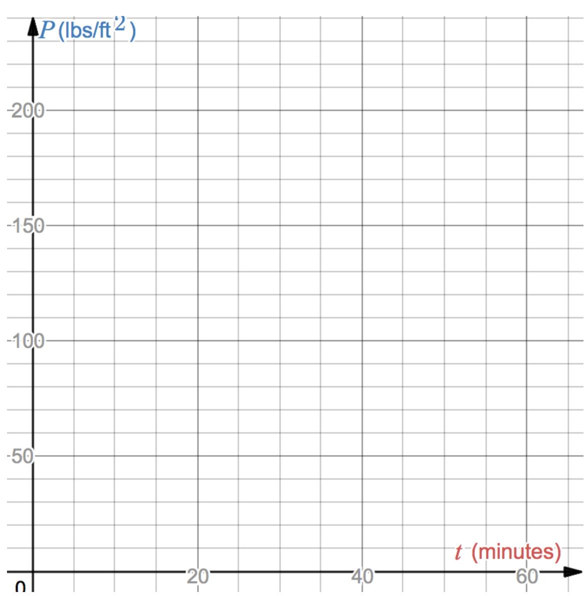
(b) Find the angle between $0°$ and $360°$ (but not $240°$) that has the same sine as $240°$.

6. (Points: 12) The pressure, P (in lbs/ft^2), in a pipe varies over time. Three times an hour, the pressure oscillates from a low of 90 to a high of 230 and then back to a low of 90. The pressure at $t = 0$ is 90.

(a) Graph $P = f(t)$, where t is time in minutes.

(b) Find a possible formula for $P = f(t)$.

(c) Using your graph from part (a) $P = f(t)$ for $0 \leq t \leq 20$, estimate when the pressure first equals 125 lbs/ft^2.

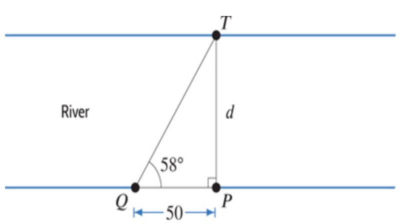


7. (Points: 8) If $\cos(\alpha) = -\sqrt{3}/5$ and α is in the third quadrant,

   (a) find the exact value for $\sin(\alpha)$,

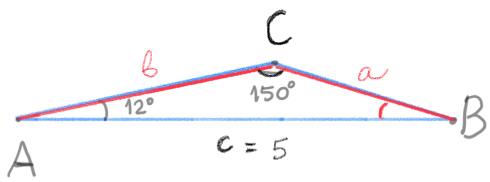
   (b) find the exact value for $\tan(\alpha)$.

8. (Points: 6) A surveyor must measure the distance between the two banks of a straight river. She sights a tree at point T on the opposite bank of the river and drives a stake into the ground (at point P) directly across from the tree. Then she walks 50 meters upstream and places a stake at point Q. She measures angle PQT and finds that it is $58°$. Find the width of the river.

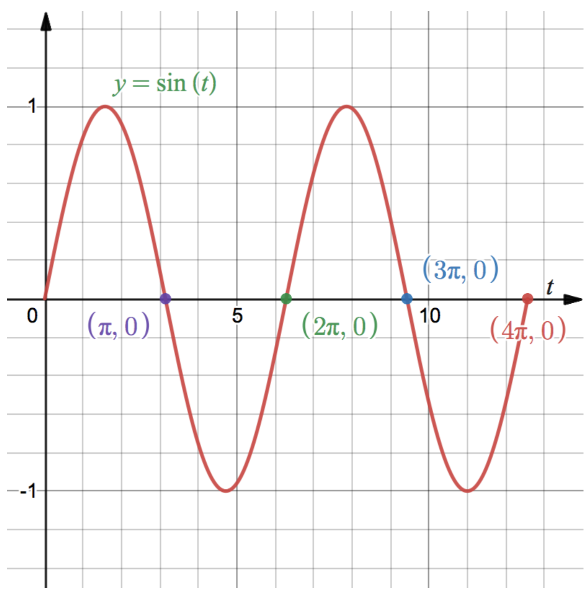


9. (Points: 8) Find the missing sides, a, b, and angle B.

$A = 12°,\ C = 150°,\ c = 5.$



10. (Points: 8) Use the graph to approximate all solutions to the equation $\sin(t) = \sqrt{2}/2$ on $0 \le t \le 4\pi$.



11. (Points: 5) Decompose the function

$$f(x) = 5\sqrt{x+3}$$

into a composition of two new functions u and v, where v is the inside function, that is $f(x) = u(v(x))$, so that $u(x) \neq x$ and $v(x) \neq x$.

12. (Points: 12) Let $P = f(t) = 37.8(1.044)^t$ be the population of a town (in thousands) in year t.

(a) Evaluate $f(50)$. Describe in words what this quantity tells you.

(b) Find a formula for $f^{-1}(P)$ in terms of P.

(c) Evaluate $f^{-1}(50)$. Describe in words what this quantity tells you.

Exponential and Logarithm Formulas

Exponential Function: $y = ab^x$

Simple Interest: $P(t) = P_0(1 + r)^t$

Compound Interest: $P(t) = P_0(1 + \frac{r}{n})^{nt}$

Continuous Growth: $P(t) = P_0 e^{rt}$

Half-life: $Q(t) = Q_0(\frac{1}{2})^{\frac{t}{T_h}}$

Doubling time: $Q(t) = Q_0 2^{\frac{t}{T_d}}$

Logarithms: $b^x = M \Leftrightarrow \log_b(M) = x$

Natural Logarithm: $\ln(x) = \log_e(x)$

Common Logartithm: $\log(x) = \log_{10}(x)$

Trigonometry

1 radian $= \frac{180}{\pi}$ degrees

1 degree $= \frac{\pi}{180}$ radians

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identity:

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta) \qquad 1 + \cot^2(\theta) = \csc^2(\theta)$

Arc Length: $s = r\theta$

Sinusoidal Functions:

$f(x) = A\sin(Bx) + k \qquad g(x) = A\cos(Bx) + k$

Period: $P = \dfrac{2\pi}{B}$

Even-Odd Identities:

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

Other Identities:

$\sin(\theta) = \sin(180° - \theta)$

$\cos(\theta) = -\cos(180° - \theta)$

$\tan(\theta) = -\tan(180° - \theta)$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

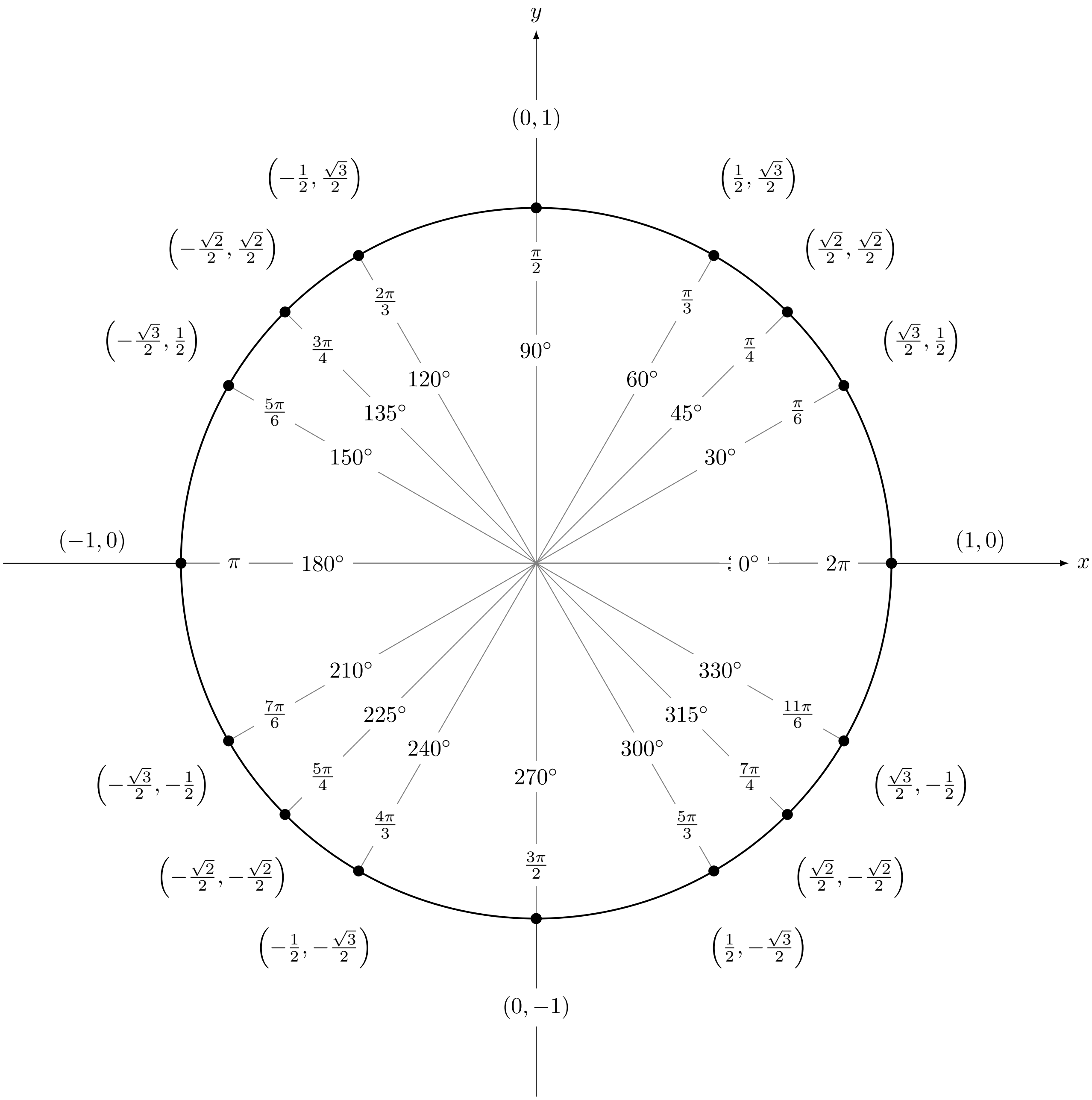
Law of Sines: $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b} = \dfrac{\sin(C)}{c}$

Inverse Trig:

$\theta = \cos^{-1} y$ provided that $y = \cos\theta$ and $0 \le \theta \le \pi$.

$\theta = \sin^{-1} y$ provided that $y = \sin\theta$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$\theta = \tan^{-1} y$ provided that $y = \tan\theta$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.



y
x
$(0,1)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$(1,0)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$(0,-1)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$(-1,0)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$ 45°
$\frac{\pi}{6}$ 30°
0° 2π
$\frac{11\pi}{6}$ 330°
$\frac{7\pi}{4}$ 315°
$\frac{5\pi}{3}$ 300°
$\frac{3\pi}{2}$ 270°
$\frac{4\pi}{3}$ 240°
$\frac{5\pi}{4}$ 225°
$\frac{7\pi}{6}$ 210°
π 180°
$\frac{5\pi}{6}$ 150°
$\frac{3\pi}{4}$ 135°
$\frac{2\pi}{3}$ 120°